PERGAMON

International Journal of Solids and Structures 36 (1999) 3349–3373

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

Interacting circular inclusions in antiplane piezoelectricity

C. K. Chao*, K. J. Chang

Department of Mechanical Engineering, National Taiwan University of Science and Technology, Taipei, Taiwan, 106, R.O.C.

Received 12 November 1997; in revised form 17 April 1998

Abstract

The problem of multiple piezoelectric circular inclusions, which are perfectly bonded to a piezoelectric matrix, is analyzed in the framework of linear piezoelectricity. Both the matrix and the inclusions are assumed to possess the symmetry of a hexagonal crystal in the 6 mm class and subject to electromechanical loadings (singularities) which produce in-plane electric fields and out-of-plane displacement. Based upon the complex variable theory and the method of successive approximations, the solution of electric field and displacement field either in the inclusions or in the matrix is expressed in terms of explicit series form. Stress and electric field concentrations are studied in detail which are dependent on the mismatch in the material constants, the distance between two circular inclusions, and the magnitude of electromechanical loadings. It is shown that, when the two inclusions approach each other, the oscillatory behavior of the stress and electric field can be induced in the inclusion as the matrix and the inclusions are poled in the opposite directions. This important phenomenon can be utilized to build a very sensitive sensor in a piezoelectric composite material system. The present derived solution can also be applied to the inclusion problem with straight boundaries. The problem associated with three-material media under electromechanical sources is also considered. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Piezoelectric inclusions; Intelligent materials; Electromechanical loadings; Successive approximations; Analytical continuation

1. Introduction

The development of piezoelectric materials or structures has been made by the research community in recent years. It is well known that piezoelectric materials will undergo deformation when subjected to an electric field and produce an electric field when deformed. Due to this intrinsic coupling phenomenon, piezoelectric materials or structures are applied widely to the design and implementation of buildings, bridges, ships, advanced aircraft, launch vehicles. These intelligent

* Corresponding author. E-mail: ckchao@mail.ntust.edu.tw

PII: S0020–7683(98)00148–6

materials provide the responsiveness by building the sensing and response functions into the material itself such as shape memory alloys, shape memory polymers, electrorheological fluids and optic fibers. Among several piezoelectric materials, ceramics are commonly used in many engineering applications such as lead zirconate titanate (PZT), which are subjected to a poling process that induces piezoelectricity and anisotropy in the material. It is known that if one takes the plane normal to the poling direction as the plane of interest, only the out-of-plane deformations and the inplane electric fields are induced. This will simplify the mathematical formulation of the problem by taking only a single displacement potential and a single electric potential as two independent variables. Studies on the problem of antiplane piezoelectricity in the presence of defects such as cracks, dislocations and inclusions have been recently investigated by various authors (Pak, 1990a, b; Zhang and Hack, 1992; Honein et al., 1995; Zhang and Tong, 1996; Zhong and Meguid, 1997). In this paper, however, a generalized and mathematically rigorous model is developed to treat the problem of multiple inclusions interacted with a host intelligent material subjected to antiplane shear and inplane electric field. The proposed method is based upon the method of analytical continuation and the technique of successive approximations which allow us to express the solutions as a rapidly convergent series. This method has a clear advantage in deriving the solution to the heterogeneous problem in terms of the solution to the corresponding homogeneous problem that was termed 'heterogenization' by Honein et al. (1992a, b). Furthermore, the present proposed method can be applied not only for the inclusion problem with circular boundaries but also for the inclusion problem with straight boundaries. It should be mentioned that the problem with two piezoelectric inclusions has been solved by Honein et al. (1995). In their paper, based upon the Moebius transformation, universal formulae are derived for the electromechanical field at the point of contact of two piezoelectric inclusions. However, the emphasis of this work is not only on demonstrating a novel and elegant method of solving boundary value problems but also on the understanding of interesting electromechanical coupling behaviors between two neighboring piezoelectric inclusions that have not been studied previously.

In the following study, a complex representation of antiplane piezoelectricity is provided in Section 2 and the solution of two piezoelectric circular inclusions is provided in Section 3. In Section 4, we will examine the stress and electric field concentrations due to the presence of inclusions which are embedded in an infinite matrix and in a half-plane matrix. A problem associated with three-material media under electromechanical sources is also studied. Finally, Section 5 concludes the article. The present work would be helpful in understanding the effects of second-phase particles and voids in piezoelectric materials and in designing piezoelectric composites to reduce the problem of dielectric breakdowns that frequently occur during a poling process.

2. A complex representation of antiplane piezoelectricity

Consider an intelligent material system composed of a host material matrix S and two circular inclusion S_j $(j = 1, 2)$ where singularities (or loads) are in the matrix (Fig. 1). Each circular inclusions has different material properties from those of the matrix, but they are assumed to have the same material orientation with x_3 in the poling direction. In a class of piezoelectric materials capable of undergoing out-of-plane displacement u_3 and in-plane electric fields ϕ, the governing field equations can be simplified to

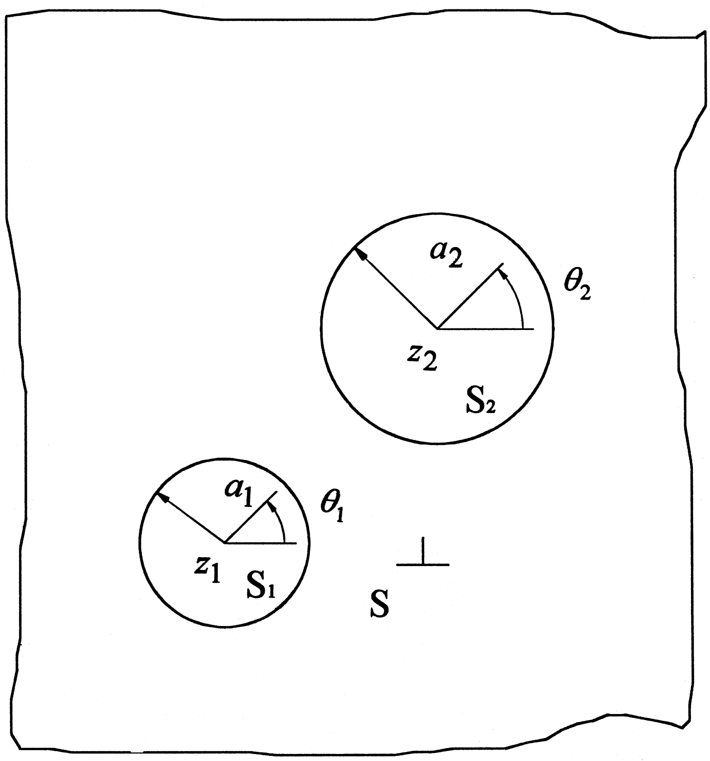

Fig. 1. Two piezoelectric inclusions perfectly bonded to an intelligent material.

$$c_{44}\nabla^2 u_3 + e_{15}\nabla^2\phi = 0 \tag{1}$$

$$e_{15}\nabla^2 u_3 - \varepsilon_{11}\nabla^2\phi = 0 \tag{2}$$

where ∇^2 is the two-dimensional Laplacian operator. c_{44} is the elastic modulus, ε_{11} is the dielectric constant, and e_{15} is the piezoelectric constant. Equations (1) and (2) are satisfied if

$$\nabla^2 u_3 = 0 \quad \text{and} \quad \nabla^2\phi = 0 \tag{3}$$

The only non vanishing components of the stress field, the electric field and the electric displacement are given by

$$\sigma_{13} = \sigma_{31} = c_{44}\frac{\partial u_3}{\partial x_1} + e_{15}\frac{\partial\phi}{\partial x_1} \tag{4}$$

$$\sigma_{23} = \sigma_{32} = c_{44}\frac{\partial u_3}{\partial x_2} + e_{15}\frac{\partial\phi}{\partial x_2} \tag{5}$$

$$E_1 = -\frac{\partial\phi}{\partial x_1} \tag{6}$$

$$E_2 = -\frac{\partial \phi}{\partial x_2} \tag{7}$$

$$D_1 = e_{15}\frac{\partial u_3}{\partial x_1} - \varepsilon_{11}\frac{\partial \phi}{\partial x_1} \tag{8}$$

$$D_2 = e_{15}\frac{\partial u_3}{\partial x_2} - \varepsilon_{11}\frac{\partial \phi}{\partial x_2} \tag{9}$$

In order to formulate the boundary value problem, it is convenient to use a complex representation for u_3 and ϕ which are grouped as a vector

$$\begin{Bmatrix} u_3 \\ \phi \end{Bmatrix} = \mathrm{Re}\,[U] \tag{10}$$

where Re denotes the real part of a complex function and U is the complex generalized displacement with two components being analytic functions. The components of the stress and electric displacement are related to the complex generalized displacement by

$$\begin{Bmatrix} \sigma_{31} - i\sigma_{32} \\ D_1 - iD_2 \end{Bmatrix} = \begin{bmatrix} c_{44} & e_{15} \\ e_{15} & -\varepsilon_{11} \end{bmatrix} U' = CU' \tag{11}$$

where prime indicates differentiation with respect to the complex variable $z = x_1 + ix_2$. In this study the interface boundary conditions are expressed in terms of the complex function U rather than its derivative U'. For this purpose, one may take an integration of the traction t and normal electric displacement D_n as

$$\int \begin{Bmatrix} t \\ D_n \end{Bmatrix} \mathrm{d}s = \mathrm{Im}\,[CU] \tag{12}$$

where $[t \quad D_n]^T$ is referred to as the generalized traction and Im denotes the imaginary part of a complex function.

3. Solutions of two piezoelectric circular inclusions

Consider two circular inclusions centered at z_j, of different arbitrary radii a_j, and of different material constants, which are perfectly bonded to the matrix of infinite extent (see Fig. 1). Our purpose is to determine the complex generalized displacements $U(z)$ and $U_j(z)$ $(j = 1, 2)$ where all the singularities (loads) are in the matrix. Before solving the problem with two circular inclusions, we first seek the solution of a single inclusion occupying a region, say S_1, as the form

$$U(z) = U_0(z) + \underset{\sim}{U}(z), \quad z \in S \tag{13}$$

$$U_1(z) = \underset{\sim}{U}_1(z), \quad z \in S_1 \tag{14}$$

where $U_0(z)$ represents the solution corresponding to the homogeneous media which is holomorphic in the entire domain except for some singular points. $\underset{\sim}{U}(z)$ (or $\underset{\sim}{U}_1(z)$) is the solution corresponding to the perturbed field of matrix (or inclusion) which is holomorphic in the region S (or S_1). By applying the continuity conditions Re $[U]$ = Re $[U_1]$ and Im $[CU]$ = Im $[C_1U_1]$ along the interface, and using the technique of analytical continuation (Muskhelishvili, 1953), one may obtain the result corresponding to the problem with a single inclusion as (Chao and Chiang, 1996)

$$U(z) = U_0(z)+\alpha_1\overline{U_0(A_1z)}, \quad z\in S \tag{15}$$

$$U_1(z) = (I+\alpha_1)U_0(z), \quad z\in S_1 \tag{16}$$

with

$$\alpha_1 = (C+C_1)^{-1}(C-C_1) \tag{17}$$

where A_1z stands for the transformation function defined as $A_1z = a_1^2/(\bar{z}-\bar{z}_1)+z_1$ and I is an 2×2 identity matrix. The overbar denotes the conjugate of a complex function. As to the problem containing two circular inclusions, the expressions (15) and (16) are certainly not suitable for this problem since the continuity conditions associated with the second inclusion are not satisfied. In order to satisfy the interface conditions across the boundary of this second inclusion, the solutions (15) and (16) are now modified to the following expressions:

$$U(z) = F_0(z)+F(z), \quad z\in S \tag{18}$$

$$U_1(z) = (I+\alpha_1)U_0(z)+F(z), \quad z\in S_1 \tag{19}$$

$$U_2(z) = F_0(z)+G(z), \quad z\in S_2 \tag{20}$$

with

$$F_0(z) = U_0(z)+\alpha_1\overline{U_0(A_1z)} \tag{21}$$

where $F(z)$ (or $G(z)$) represents the complex generalized displacement associated with the perturbed field of matrix (or inclusion) which is holomorphic in the region S (or S_2). Applying the continuity conditions along the interface $z = \sigma = z_2+a_2e^{i\theta_2}$ with the aid of (10) and (12), we have the following two equations:

$$F(\sigma)+\overline{F(\sigma)} = G(\sigma)+\overline{G(\sigma)} \tag{22}$$

$$C[F_0(\sigma)+F(\sigma)-\overline{F_0(\sigma)}-\overline{F(\sigma)}] = C_2[F_0(\sigma)+G(\sigma)-\overline{F_0(\sigma)}-\overline{G(\sigma)}] \tag{23}$$

Using the continuation theorem (Muskhelishvili, 1953), (22) and (23) enables us to define a new set of complex potentials $\Theta_j(z)$ ($j = 1, 2$), which is holomorphic in the entire domain including the interface as

$$\Theta_1(z) = F(z)-\overline{G(A_2z)} \tag{24}$$

$$\Theta_2(z) = C[F(z)-\overline{F(A_2z)}]+C_2[\overline{F_0(A_2z)}+\overline{G(A_2z)}] \tag{25}$$

for $z\in z+z_1$, and

$$\Theta_1(z) = G(z)-\overline{F(A_2z)} \tag{26}$$

$$\Theta_2(z) = C_2[F_0(z)+G(z)]+C[\overline{F(A_2z)}-F_0(z)] \tag{27}$$

for $z\in S_2$. A_2z represents the transformation function defined as $A_2z = a_2^2/(\bar{z}-\bar{z}_2)+z_2$. Since $\Theta_j(z)$

are holomorphic in the whole domain including the point at infinity, by Liouville's theorem we have $\Theta_j(z)$ = constant. However, the constant functions $\Theta_j(z)$ can be treated as a rigid body motion and can thus be assumed to be zero without loss in generality. Having this result, (24)–(27) yield the following expressions

$$\underset{\sim}{F}(z) = \alpha_2 \overline{F_0(A_2 z)} \tag{28}$$

$$\underset{\sim}{G}(z) = \alpha_2 F_0(z) \tag{29}$$

with

$$\alpha_2 = (C + C_2)^{-1}(C - C_2) \tag{30}$$

With the aid of (28) and (29), the expressions in (18)–(20) become

$$U(z) = U_0(z) + \alpha_1 \overline{U_0(A_1 z)} + \alpha_2 \overline{U_0(A_2 z)} + \alpha_2 \alpha_1 U_0(A_1 A_2 z), \quad z \in S \tag{31}$$

$$U_1(z) = U_0(z) + \alpha_1 U_0(z) + \alpha_2 \overline{U_0(A_2 z)} + \alpha_2 \alpha_1 U_0(A_1 A_2 z), \quad z \in S_1 \tag{32}$$

$$U_2(z) = U_0(z) + \alpha_1 \overline{U_0(A_1 z)} + \alpha_2 U_0(z) + \alpha_2 \alpha_1 \overline{U_0(A_1 z)}, \quad z \in S_2 \tag{33}$$

In view of (31)–(33), the continuity conditions are now satisfied at the interface along the second inclusion, but not at the interface along the first inclusion. Repeating the previous steps and obtaining the two additional terms each time, the results of which the continuity conditions are satisfied at both the interfaces along the first and second inclusions can be found as the following explicit forms

$$U(z) = U_0(z) + \sum_{n=1}^{\infty} (\alpha_1 \alpha_2)^n U_0(M^n z) + \sum_{n=1}^{\infty} (\alpha_1 \alpha_2)^n U_0(N^n z) + \alpha_1 \sum_{n=0}^{\infty} (\alpha_1 \alpha_2)^n \overline{U_0(A_1 N^n z)}$$
$$+ \alpha_2 \sum_{n=0}^{\infty} (\alpha_1 \alpha_2)^n \overline{U_0(A_2 M^n z)}, \quad z \in S \tag{34}$$

$$U_1(z) = (I + \alpha_1) \left\{ U_0(z) + \alpha_2 \sum_{n=0}^{\infty} (\alpha_1 \alpha_2)^n \overline{U_0(A_2 M^n z)} + \sum_{n=1}^{\infty} (\alpha_1 \alpha_2)^n U_0(M^n z) \right\}, \quad z \in S_1 \tag{35}$$

$$U_2(z) = (I + \alpha_2) \left\{ U_0(z) + \alpha_1 \sum_{n=0}^{\infty} (\alpha_1 \alpha_2)^n \overline{U_0(A_1 N^n z)} + \sum_{n=1}^{\infty} (\alpha_1 \alpha_2)^n U_0(N^n z) \right\}, \quad z \in S_2 \tag{36}$$

where $M^n z = (A_1 A_2)^n z$, $N^n z = (A_2 A_1)^n z$. Equation (34)–(36) give a general series solution of the problem containing two circular piezoelectric inclusions as the corresponding homogeneous solution $U_0(z)$ is solved. It should be emphasized that, based upon the method of successive approximations described earlier, the present derived solution can be extended to the problem containing any number of piezoelectric inclusions.

4. Stress and electric field concentrations

In this section, the derived series solutions (34)–(36) are used to analyze the following examples associated with the piezoelectric problem of an infinite matrix, of a half-plane matrix, and of three-material media. In the following discussion, the material constants of matrix and two inclusions are assumed as $c_{44}^M = c_{44}^I = 3.53 \times 10^{10}$ Nm^{-2}, $\varepsilon_{11}^M = \varepsilon_{11}^I = 1.51 \times 10^{-8}$ CV^{-1}m^{-1}, $e_{15}^I = 10$ Cm^{-2} and other values are stated specifically.

4.1. Two circular piezoelectric inclusions embedded in a matrix

As our first example we consider two circular inclusions, of different radii $a_1 = a$, $a_2 = 2a$, perfectly bonded to a matrix which is subjected to a uniform state of dress τ_∞ and electric field E_∞ at infinity along the x_2-axis (see Fig. 2). The solution of the corresponding homogeneous problem is trivially given as

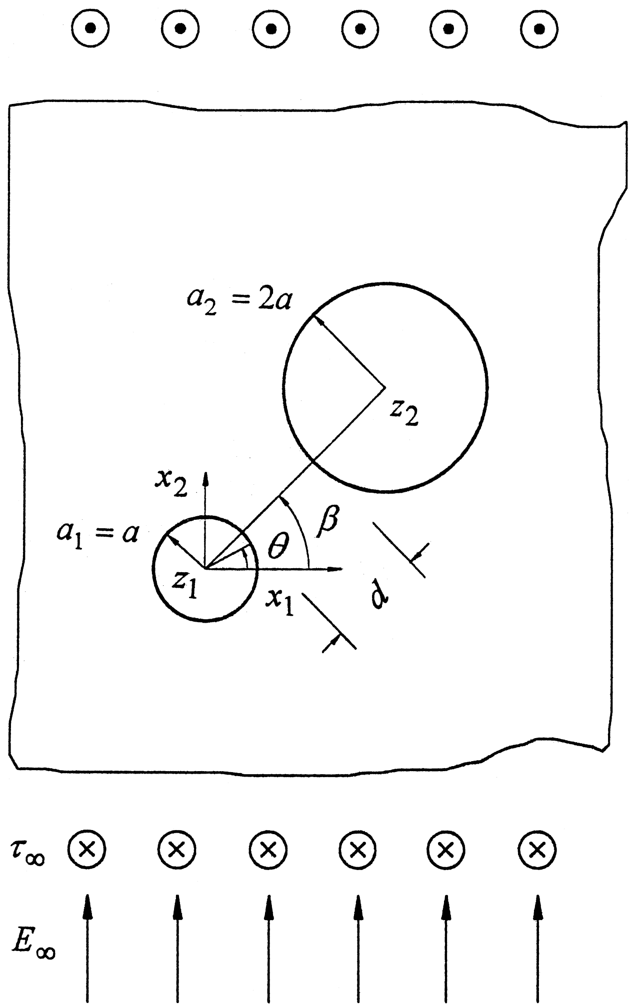

Fig. 2. Two circular inclusions embedded in a matrix subjected to a remote electromechanical load along the x_2-axis.

$$U_0(z) = \begin{Bmatrix} \dfrac{\tau_\infty + e_{15}E_\infty}{c_{44}} \\ -E_\infty \end{Bmatrix}(-iz) \tag{37}$$

Having the homogeneous solution (37), the result associated with the problem containing two circular inclusions can be immediately obtained by substituting (34)–(36) into (4)–(9). In order to examine the accuracy of the present derived solution, the stress concentration factor σ_{32}/τ_∞ in the matrix at $\theta = 0^\circ$ with $\tau_\infty = 5\times10^7$ Nm^{-2} and $E_\infty = 10^6$, 0, 10^{-6} Vm^{-1}, when the two circular inclusions are arrayed parallel to the applied loadings ($\beta = 90^\circ$) and the distance between two circular inclusions $d/a = 10$, is plotted in Fig. 3 as a function of the ratio of piezoelectric constants e^M_{15}/e^I_{15}. It is found that the calculated results displayed in Fig. 3, which are determined by summing up the first 20 terms in (34), agree very well with those of the corresponding single inclusions

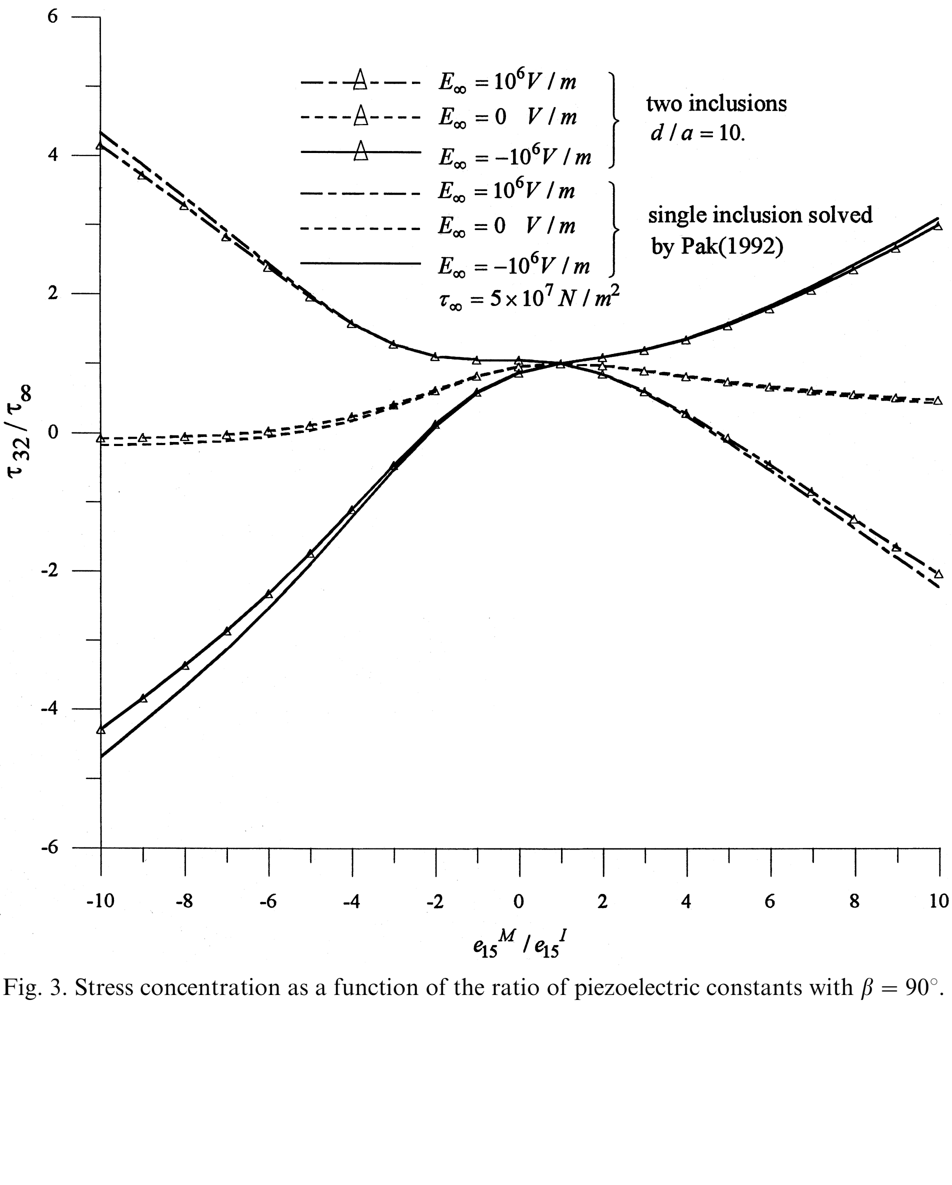

Fig. 3. Stress concentration as a function of the ratio of piezoelectric constants with $\beta = 90^\circ$.

problem provided by Pak (1992). From Fig. 4, the electric field concentration occurred at the point $\theta = 0^\circ$ is plotted as a function of the ratio of dielectric constants $\varepsilon_{11}^M/\varepsilon_{11}^I$ while letting $c_{44}^M = c_{44}^I = 3.53 \times 10^{10}$ Nm^{-2}, $e_{15}^M = e_{15}^I = 17$ Cm^{-2} and $\varepsilon_{11}^I = 1.51 \times 10^{-8}$ CV^{-1}m^{-1}. It is shown that the electric field concentration approaches two for a large $\varepsilon_{11}^M/\varepsilon_{11}^I$ as $d/a = 10$ which is consistent with the result of the corresponding single inclusion problem provided by Pak (1992). Note that all the calculated results shown in Figs 4–14 are determined by summing up the first 30 terms in (34) since a sum of the first 30 terms are checked to achieve a good accuracy with an error less than 0.1% as compared to a sum of the first 40 terms even for the case $d/a = 0.001$. When the two inclusions approach each other, both the tangential stress σ_θ and tangential electric field E_θ in the matrix along the boundary of the first inclusion are plotted in Figs 5 and 6, respectively, as the piezoelectric constants are fixed at $e_{15}^M/e_{15}^I = 3$. Note that the tangential stress increases as decreasing of the distance d/a as indicated in Fig. 5 which is basically different from the result of uniform

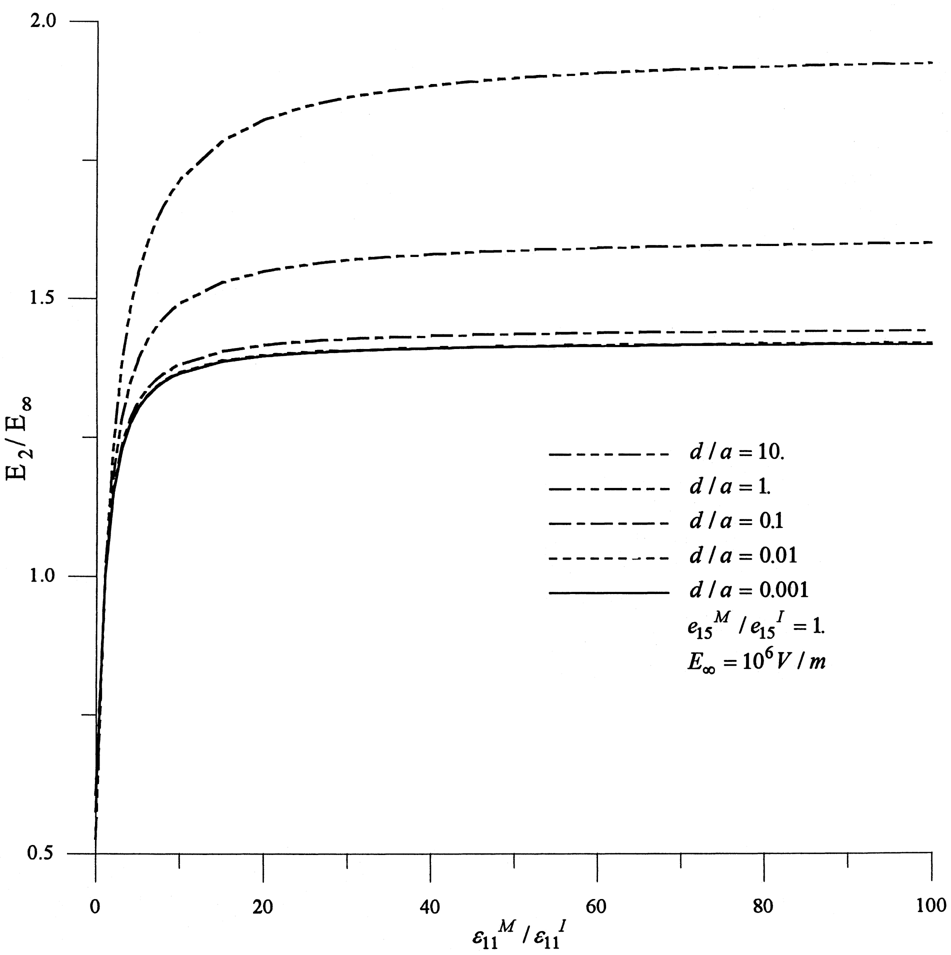

Fig. 4. Electric field concentration as a function of the ratio of dielectric constants with $e_{15}^M/e_{15}^I = 1$ and $\beta = 90^\circ$.

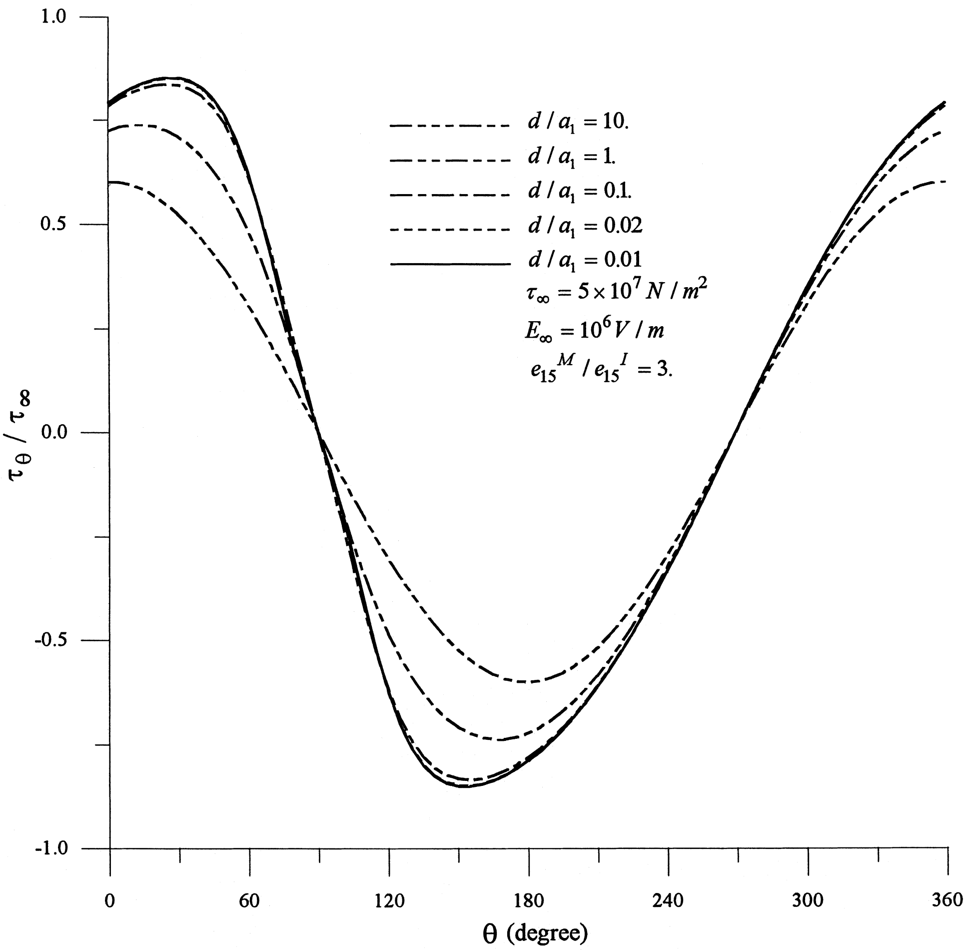

Fig. 5. Tangential stress distribution for different ratios d/a with $e_{15}^M/e_{15}^I = 3$ and $\beta = 90°$.

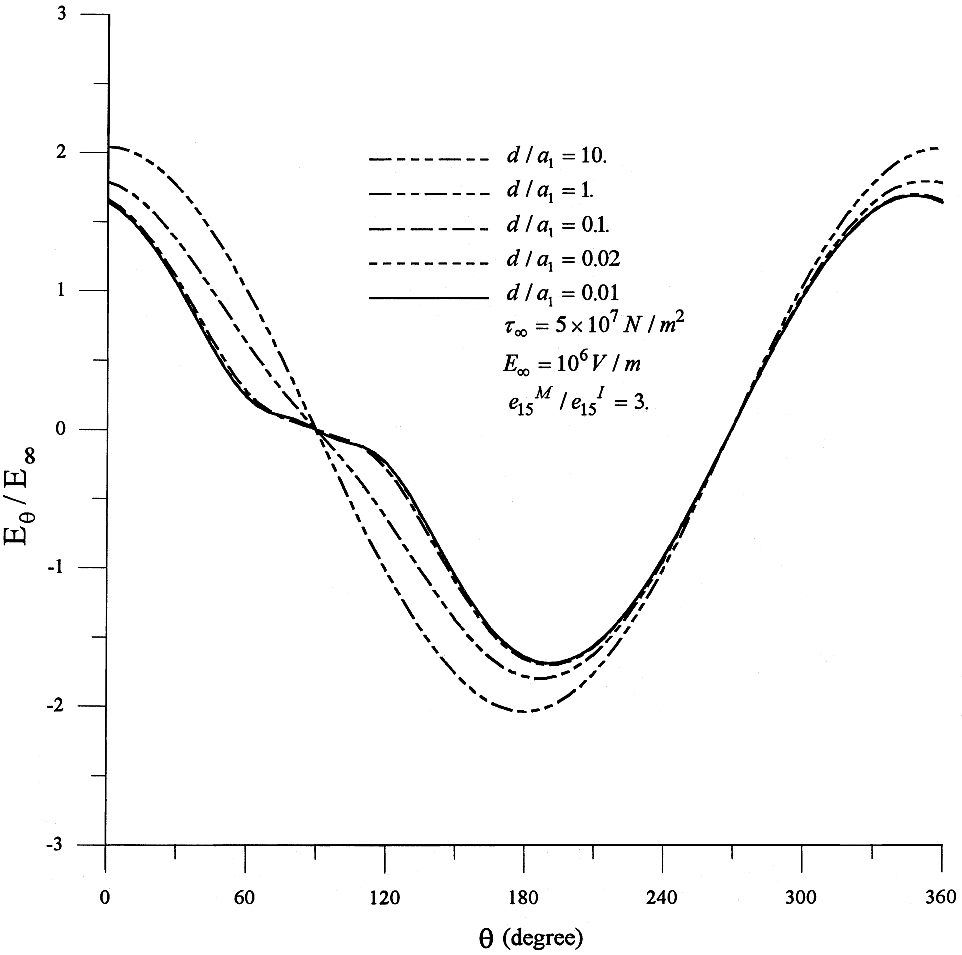

Fig. 6. Tangential electric field distribution for different ratios d/a with $e_{15}^M/e_{15}^I = 3$ and $\beta = 90°$.

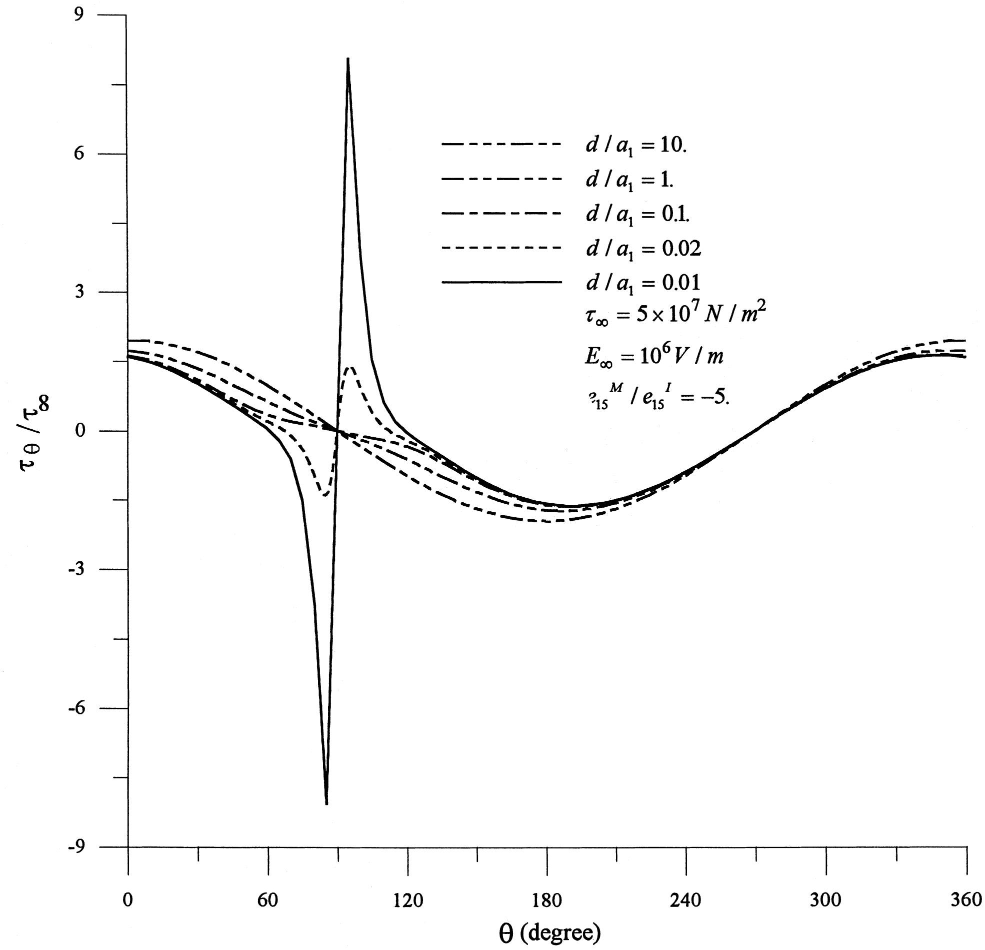

Fig. 7. Tangential stress distribution for different ratios d/a with $e_{15}^{M}/e_{15}^{I} = -5$ and $\beta = 90°$.

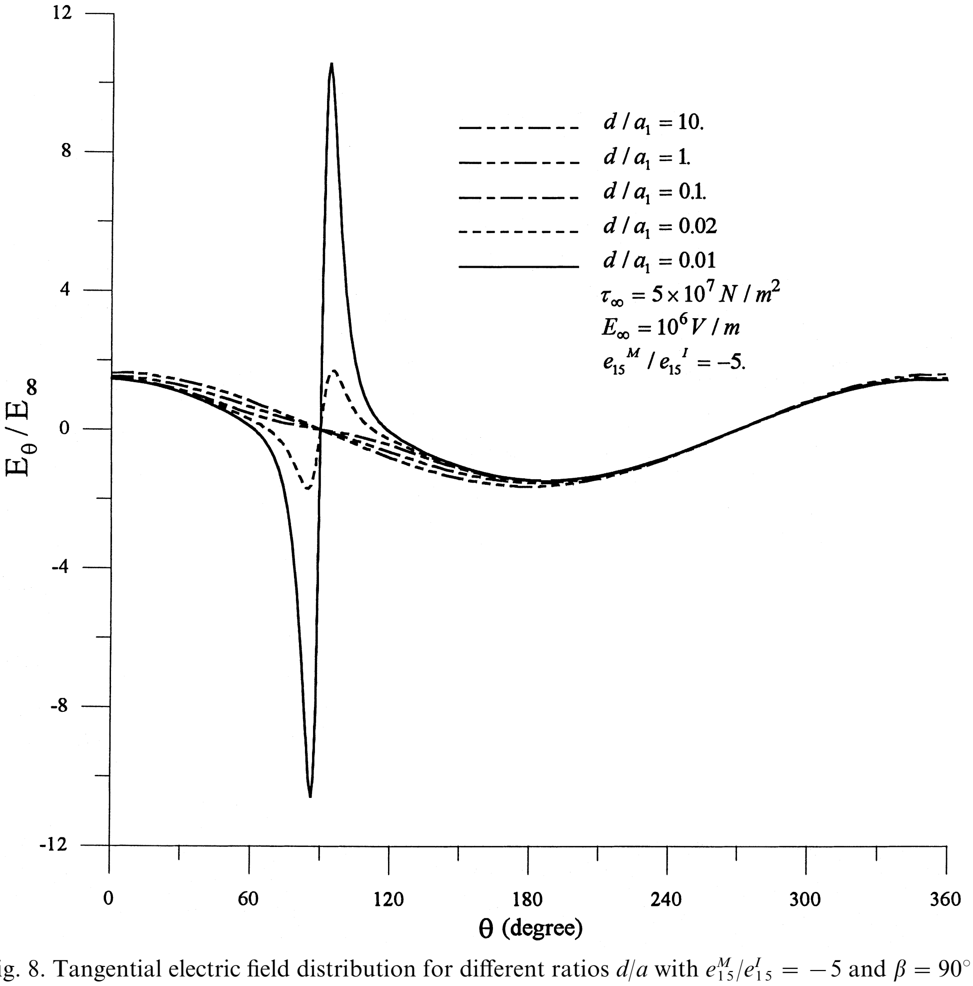

Fig. 8. Tangential electric field distribution for different ratios d/a with $e_{15}^M/e_{15}^I = -5$ and $\beta = 90°$.

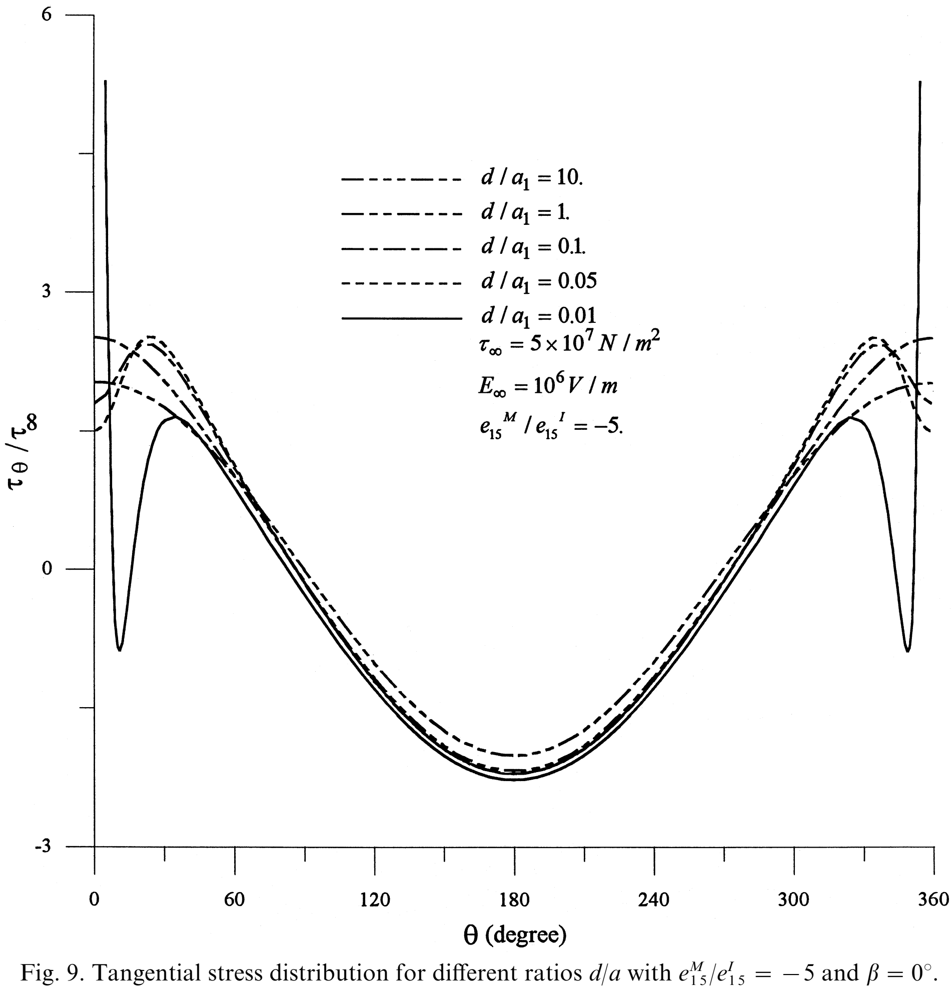

Fig. 9. Tangential stress distribution for different ratios d/a with $e_{15}^M/e_{15}^I = -5$ and $\beta = 0°$.

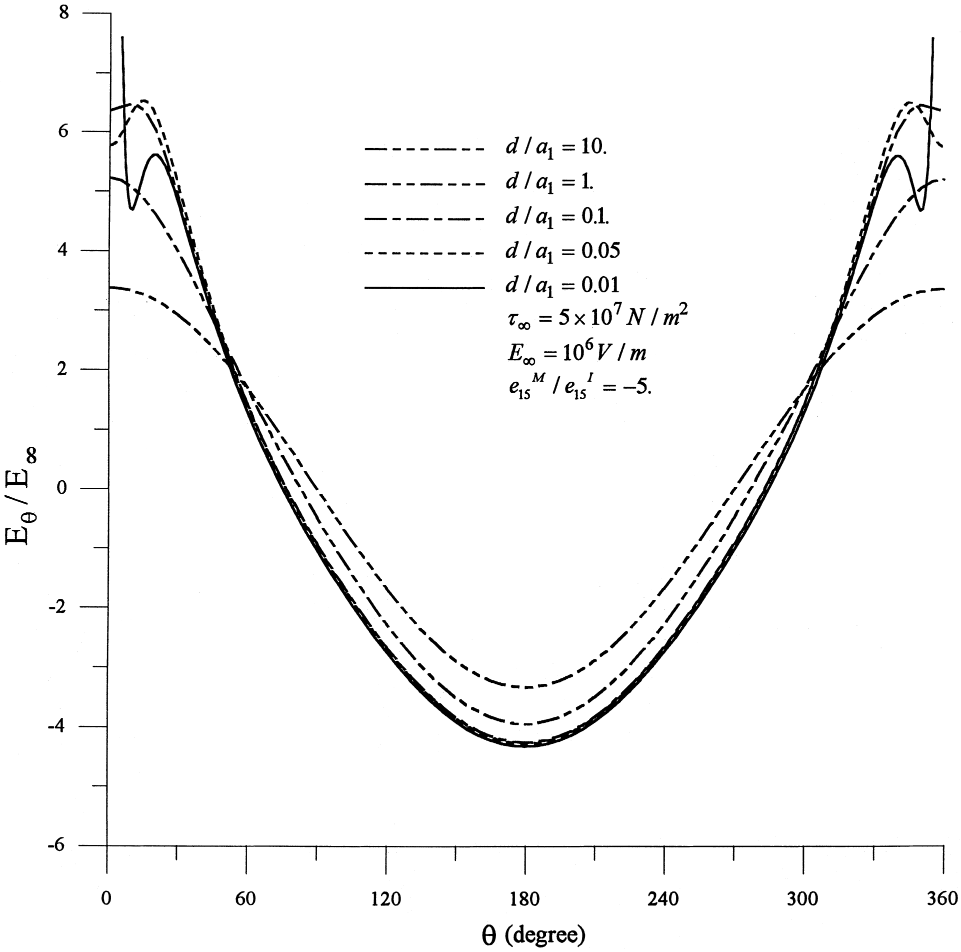

Fig. 10. Tangential electric field concentration for different ratios d/a with $e^M_{15}/e^I_{15} = -5$ and $\beta = 0°$.

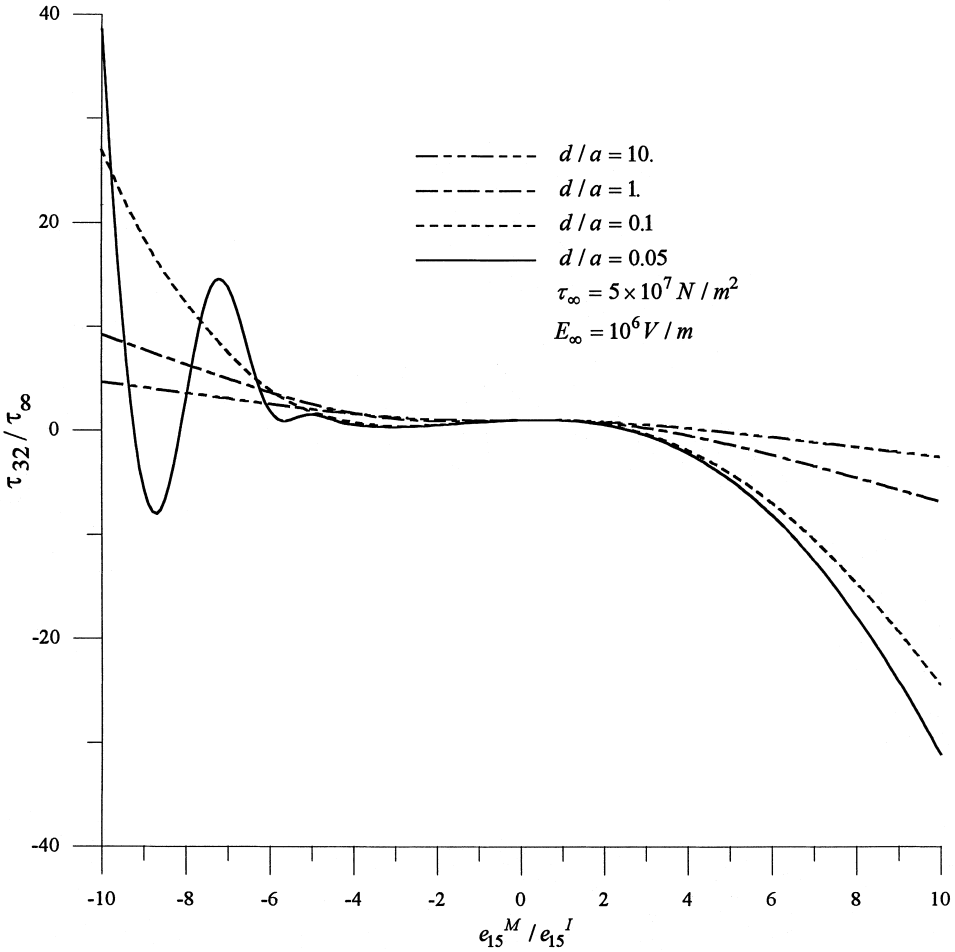

Fig. 11. Stress concentration as a function of the ratio of piezoelectric constants with $\beta = 0°$.

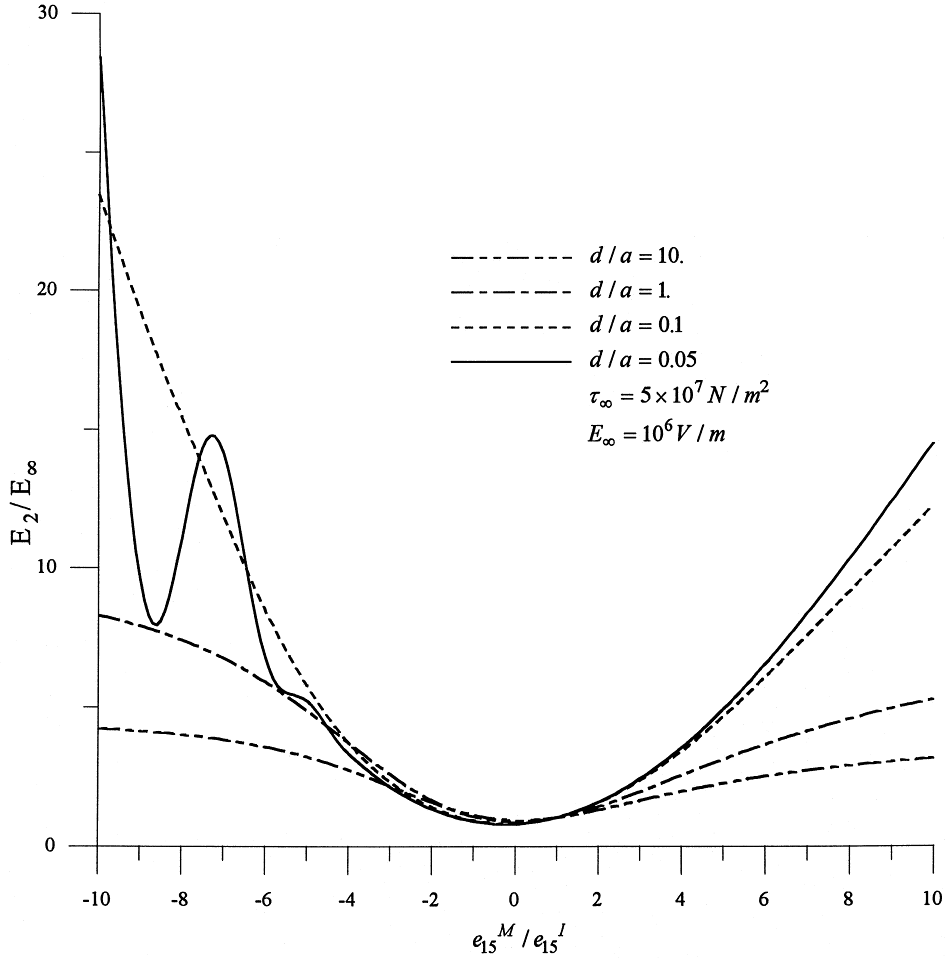

Fig. 12. Electric field concentration as a function of the ratio of piezoelectric constants with $\beta = 0°$.

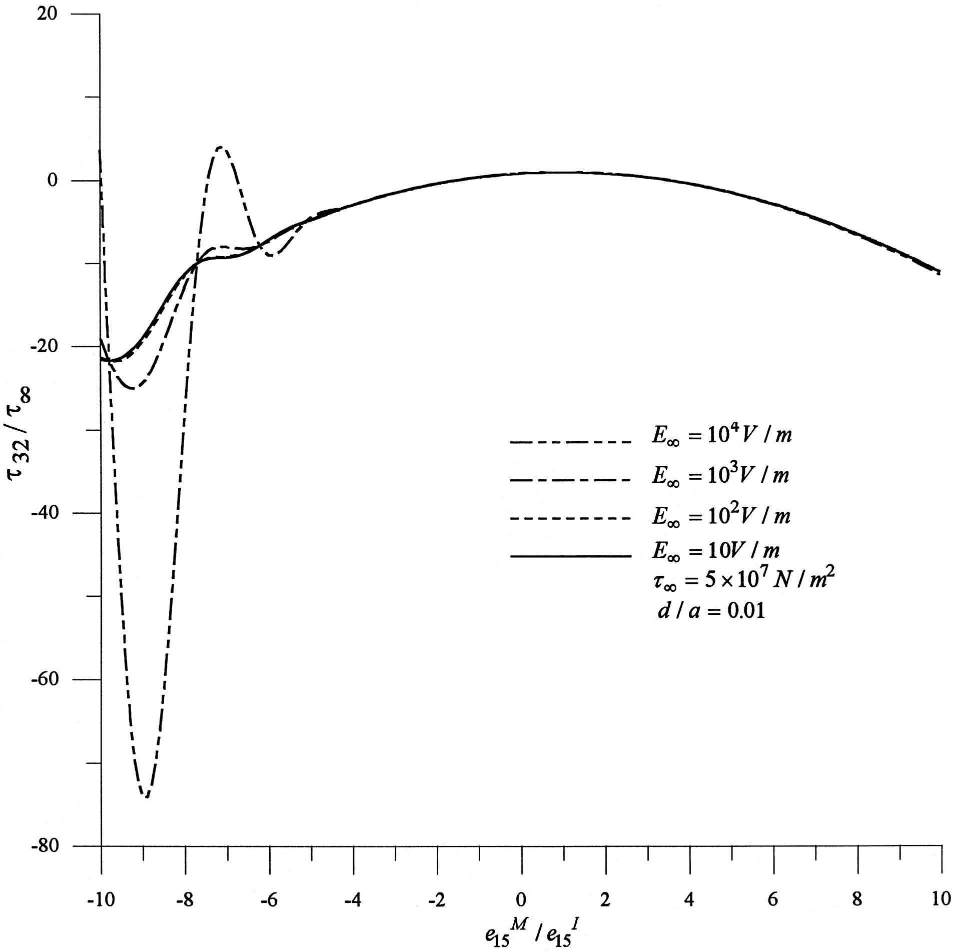

Fig. 13. Stress concentration as a function of the ratio of piezoelectric with $\beta = 0°$.

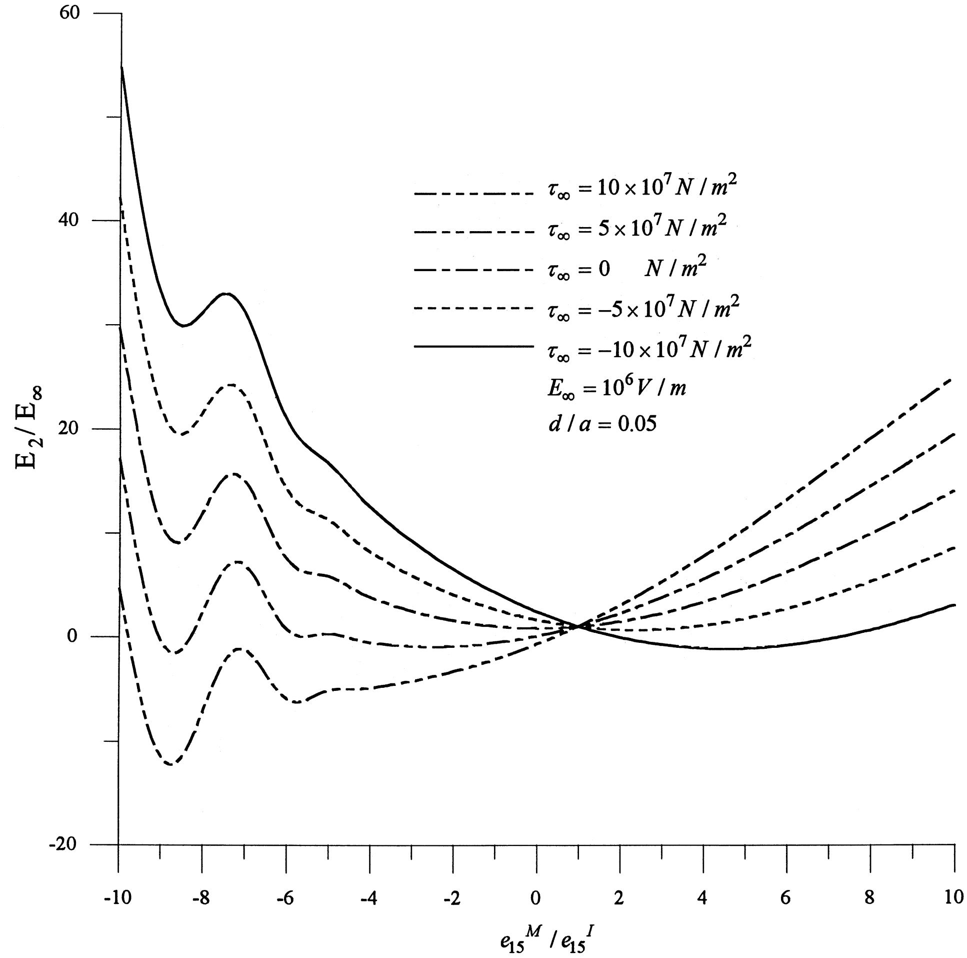

Fig. 14. Electric field concentration as a function of the ratio of piezoelectric constants with $\beta = 0°$.

stress existing in the corresponding purely elastic problem. It should be emphasized that decreasing of the distance d/a may not always result in increasing of the stress concentration for the case $e_{15}^M/e_{15}^I = 3$ as indicated in Fig. 5; it has to be matched with the rest of the material constants. It is interesting to see that, if the matrix is subjected to the reversal of the poling direction as compared to the inclusion such as the ratio of piezoelectric constants $e_{15}^M/e_{15}^I = -5$, both the stress σ_θ and the electric field E_θ experience a big jump across the point $\theta = 90°$ which is nearest to the neighboring inclusion as the distance d/a is smaller than 0.02 (see Figs 7 and 8). When the two circular piezoelectric inclusions are arrayed perpendicular to the applied loadings ($\beta = 0°$), both the stress and electric field also experience a big jump across the point $\theta = 0°$ which is nearest to the neighboring inclusions as $e_{15}^M/e_{15}^I = -5$ and $d/a \leqslant 0.05$ (see Figs 9 and 10). Variations of stress and electric field concentration occurred at the point $\theta = 0°$ with the ratio of piezoelectric constants are shown in Figs 11 and 12, respectively. Both the stress and electric field concentrations are found to be oscillatory with the ratio e_{15}^M/e_{15}^I as the distance between two neighboring inclusions

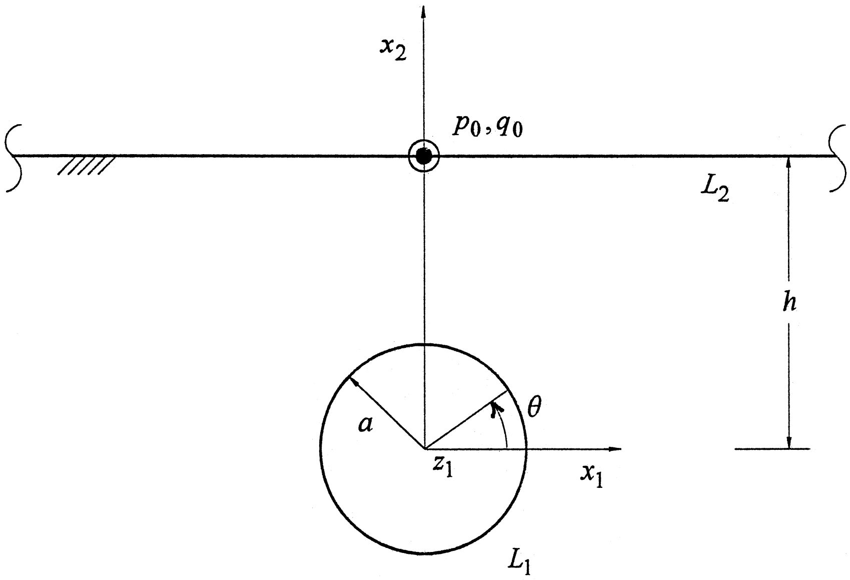

Fig. 15. A piezoelectric circular inclusion perfectly bonded to a half plane matrix.

$d/a \leqslant 0.05$ and the ratio of piezoelectric constants $e^M_{15}/e^I_{15} \leqslant -5$. This important phenomenon implies that one can build a very sensitive sensor in a piezoelectric composite material system if the relative location between two circular piezoelectric inclusions and the ratio of piezoelectric constants between the matrix and the inclusions are properly selected. Note that this phenomenon occurs for the piezoelectric matrix–piezoelectric inclusion composite system only when $e^M_{15}/e^I_{15} < -5$. i.e., the matrix and the inclusions are poled in the opposite directions. It is seen that, from Figs 11–14, both the stress and electric field concentrations are equal to one as $e^M_{15}/e^I_{15} = 1$ which are in accordance with the results of the corresponding homogeneous problem. The oscillatory feature of stress and electric field concentrations can be further intensified by increasing the magnitude of a far-field inplane electric load E_∞ and a far-field antiplane shear τ_∞, respectively, as indicated in Figs 13 and 14.

4.2. A single piezoelectric inclusion embedded in a half-plane matrix

As our second example we consider a piezoelectric inclusion perfectly embedded in a half-plane matrix which is subjected to a line force p_0 and a line charge q_0 at the surface of half-plane (see Fig. 15). The solution associated with this half-plane problem can be immediately obtained by substituting $A_2 z = \bar{z} + 2ih$ and $\alpha_2 = I$ (I is an 2×2 identity matrix) into (34) with the complex generalized displacement $U_0(z)$ being

$$U_0(z) = \lim_{x_2 \to h} \left\{ \begin{matrix} \dfrac{-\varepsilon_{11} p_0 + e_{15} q_0}{(c_{44}\varepsilon_{11} + e_{15}^2)} \\ -\dfrac{e_{15} p_0 + c_{44} q_0}{(c_{44}\varepsilon_{11} + e_{15}^2)} \end{matrix} \right\} \log(z - ix_2) \tag{38}$$

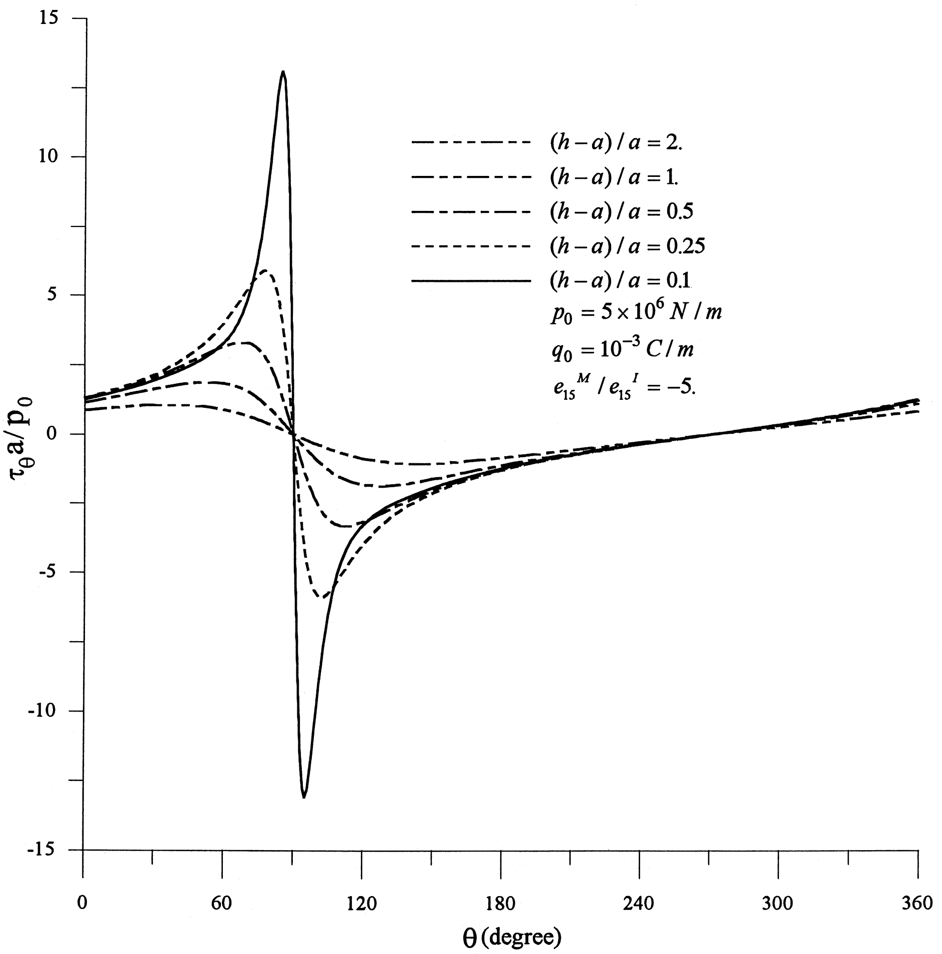

Fig. 16. Tangential stress distribution for different ratios $(h–a)/a$ with $e^M_{15}/e^I_{15} = -5$.

The calculated results of the present half-plane problem are obtained from the series solution up to the first 40 terms in (34) with an error less than 0.1% as compared to a sum of the first 50 terms. Figures 16 and 17 illustrate the tangential shear stress and electric field along the boundary of the piezoelectric inclusion for different positions of the inclusion relative to the free surface. Similar to the result of the previous problem, both the stress and electric field exhibit a big jump at the point $\theta = 90^\circ$ which is nearest to the free surface as the inclusion approaches the half plane. Keeping in mind that this phenomenon always occurs for the piezoelectric matrix–piezoelectric inclusion composite system only when the matrix and the inclusion are poled in the opposite directions.

4.3. *A piezoelectric matrix sandwiched between two piezoelectric inclusions*

As our third example we consider the problem of three-material media with piezoelectric matrix sandwiched between two piezoelectric inclusions where the matrix is subjected to a point force p_0

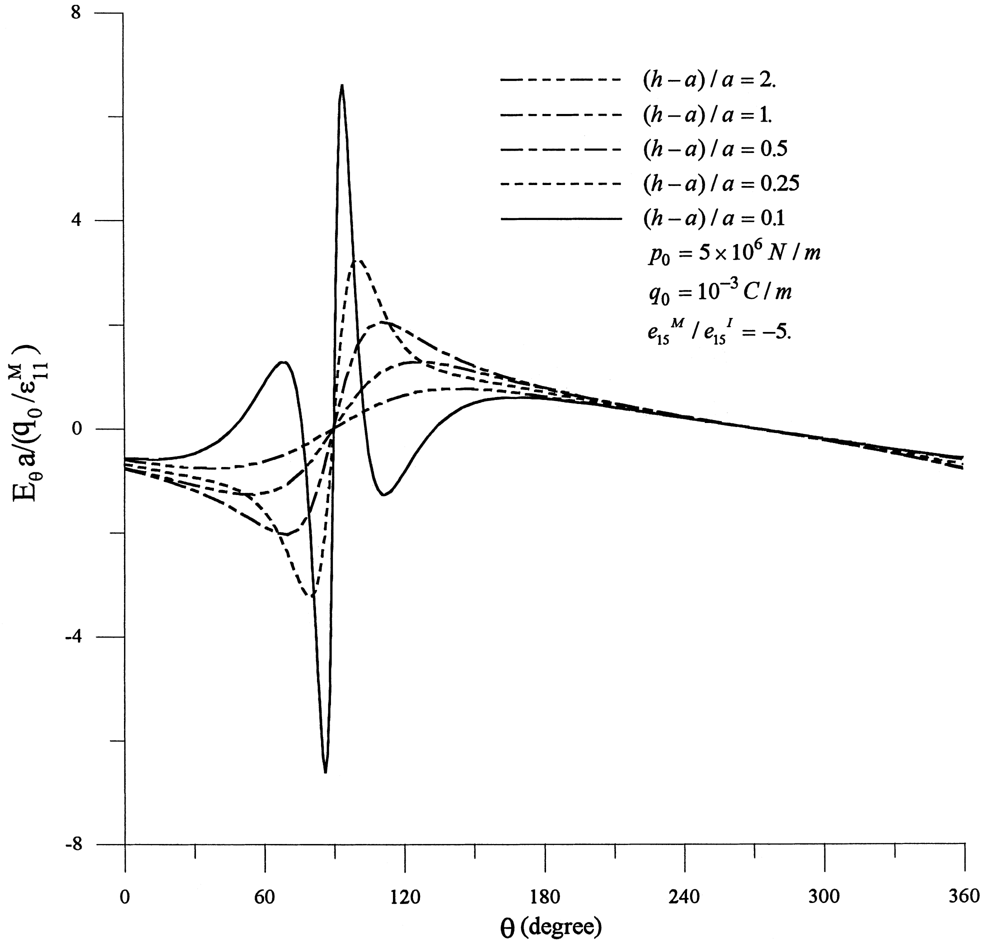

Fig. 17. Tangential electric field distribution for different ratios $(h–a)/a$ with $e_{15}^M/e_{15}^I = -5$.

and a point charge q_0 (see Fig. 18). The solution pertaining to this problem can be directly obtained by substituting $A_1 z = z+2ih$ and $A_2 z = z-2ih$ into (34) with the complex generalized displacement $U_0(z)$ being

$$U_0(z) = \begin{bmatrix} \dfrac{-\varepsilon_{11}p_0 + e_{15}q_0}{(c_{44}\varepsilon_{11} + e_{15}^2)} \\ -\dfrac{e_{15}p_0 + c_{44}q_0}{(c_{44}\varepsilon_{11} + e_{15}^2)} \end{bmatrix} \log z \tag{39}$$

The presented results are determined for the series solution up to the first 50 terms in (34) with an error less than 0.1% as compared to a sum of the first 60 terms. Figures 19 and 20 display the normal shear stress σ_{32} and the electric field E_2 at the point $x_1 = 0$, $x_2 = h$ in the matrix along the interface for different ratios of the piezoelectric constants. Both the stress and electric field are

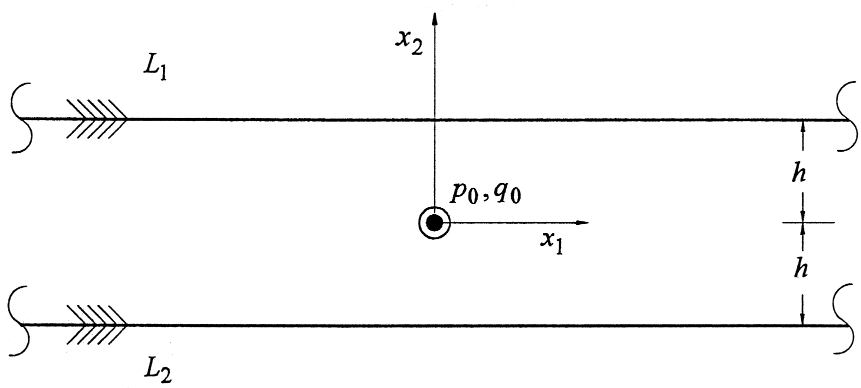

Fig. 18. A piezoelectric matrix sandwiched between two piezoelectric inclusions.

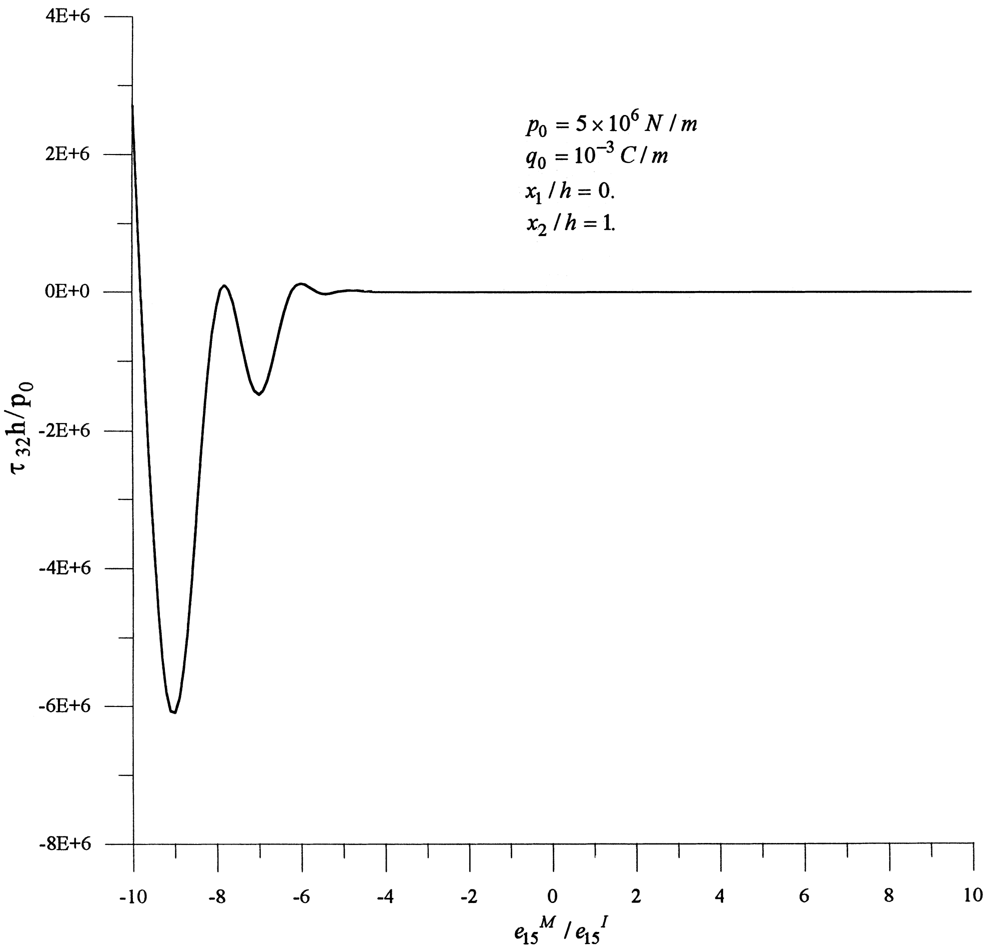

Fig. 19. Shear stress along the interface as a function of the ratio of piezoelectric constants.

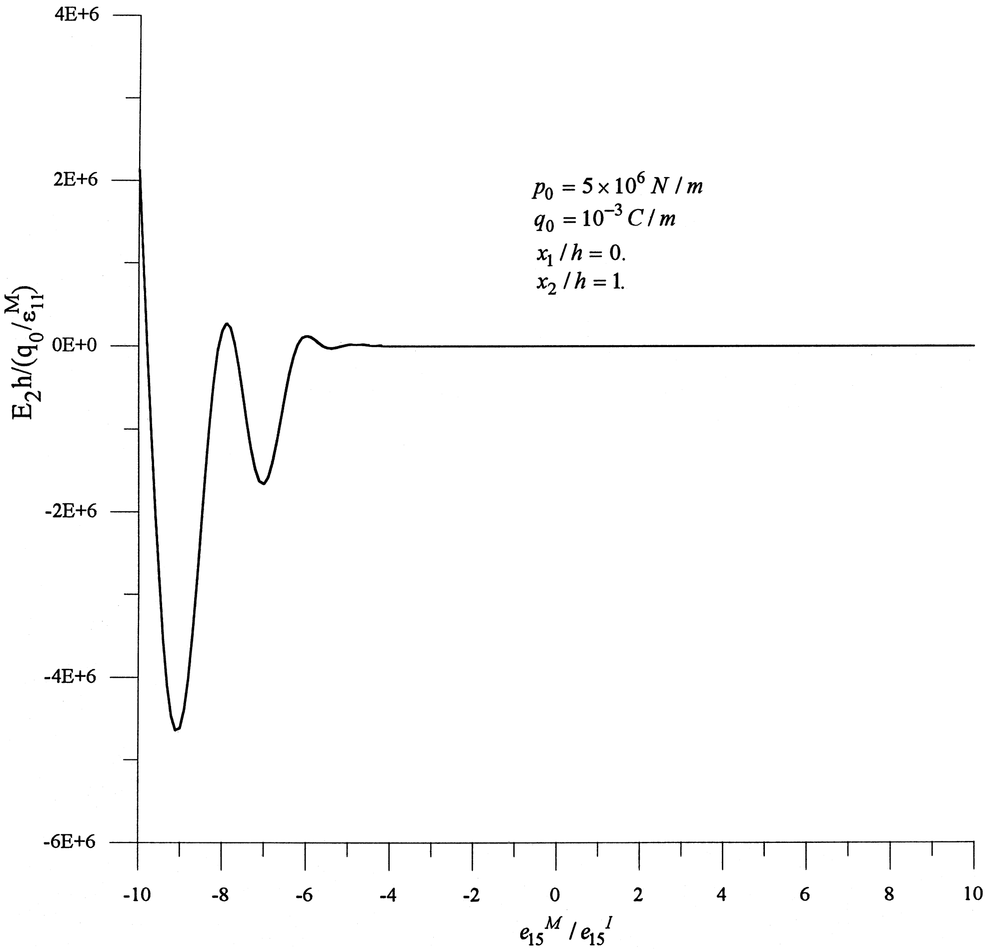

Fig. 20. Electric field along the interface as a function of the ratio of piezoelectric constants.

found to be oscillatory with the ratio of piezoelectric constants as $e_{15}^M/e_{15}^I < -5$. It is then concluded that a very sensitive sensor built in the piezoelectric sandwiched composite system can be achieved if one fabricates piezoelectric materials with $c_{44}^M = c_{44}^I$ and $\varepsilon_{11}^M = \varepsilon_{11}^I$, and a higher piezoelectric constant in the matrix but opposite in sign than that in the inclusion.

5. Concluding remarks

The problem of two piezoelectric inclusions of different radii and of different material properties, perfectly bonded to a host intelligent material matrix of infinite extent is analyzed in the framework of linear piezoelectricity. Based upon the method of analytical continuation and the technique of successive approximations, the solution was obtained as a transformation on the solution to the corresponding homogeneous problem. It was shown that both stress and electric field concentrations are dependent on the mismatch in the material constants, the relative location between two piezoelectric inclusions and the magnitude of electromechanical loadings. When the two piezoelectric inclusions approach each other, a high electric field concentration can be induced if the matrix and the inclusions are poled in the opposite directions. Similar phenomenon can also be observed for the problem with a half-plane matrix and three-material media. This important of phenomenon enables us to build a very sensitive sensor in the piezoelectric matrix–piezoelectric inclusion composite system.

References

Chao, C.K. Chiang, T.F., 1996. Antiplane interaction of an anisotropic elliptic inclusion with an arbitrarily oriented crack. Int. J. Fract 75, 229–245.

Honein, E., Honein, T., Herrmann, G., 1992a. On two circular inclusions in harmonic problems. Quarterly of Applied Mathematics L (3), 479–499.

Honein, E., Honein, T., Herrmann, G., 1992b. Further aspects of the elastic field for two circular inclusions in antiplane elatstostatics. J. Appl. Mech 59, 774–779.

Honein, E., Honein, T., Herrmann, G., 1995. On the interaction of two piezoelectric fibers embedded in an intelligent material. Journal of Intelligent Material Systems and Structures 6, 229–236.

Muskhelishvili, E.I., 1953. Some Basic Problems of the Mathematical Theory of Elasticity. Noordhoff, Groningen.

Pak, Y.E., 1990a. Crack extension force in a piezoelectric material. J. Appl. Mech 57, 647–655.

Pak, Y.E., 1990b. Force on a piezoelectric screw dislocation. J. Appl. Mech 57, 863–869.

Pak, Y.E., 1992. Circular inclusion problem in antiplane piezoelectricity. Int. J. Solids Structures 29, 2403–2419.

Zhang, T.Y., Hack, J.E., 1992. Mode III cracks in piezoelectric materials. J. Appl. Phys. 71, 5865–5870.

Zhang, T.Y., Tong, P., 1996. Fracture mechanics for a mode III crack in a piezoelectric material. Int. J. Solids Structures 33, 343–359.

Zhong, Z., Meguid, S.A., 1997. Interfacial debonding of a circular inhomogeneity in piezoelectric materials. Int. J. Solids Structures 34, 1965–1984.